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Abstract
Finite element analysis (FEA) has been carried out on 2D carbon/carbon (C/C) laminates containing a central circular hole utilizing the ANSYS software package. The stress concentration factor obtained from the existing empirical relation is comparable with the FEA result. Utilizing the stress concentration factor and the un-notched tensile strength of the 2D carbon/carbon laminates, the notched tensile strength estimates are found to be highly conservative. Modifications are made in one of the stress fracture criteria of Whitney and Nuismer known as the point stress criterion to estimate the notched strength close to the test results.

Keywords
Finite element analysis, stress concentration factor, point stress criterion, notched tensile strength, carbon/carbon composites

1. Introduction

Numerous structural applications of composite materials require the presence of holes and cut-outs, whose effect on laminate strength is not, however, fully mastered. The stress distribution around the hole and the resulting damage progression and failure of composite laminates under loading is of obvious interest to engineers and designers. Owing to the inherent complexity and the number of factors involved in predicting the notched strength of composite laminates, several semi-empirical failure criteria have been proposed and have gained popularity because of their simplicity of application. Wu *et al.* [1] have made a brief review on fracture models for

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assessing the residual strength of notched fiber metal laminates. Concerning failure criteria, three basic types of approaches can be found in the literature: fracture mechanics models [2–5]; stress-fracture criteria [6–9] and progressive damage models [10, 11]. Hallet and Winsom [11] have followed the cohesive zone technique to model the damage in each ply and between plies, rather than simply replacing all damage with a single equivalent crack. They found excellent agreement between experimental and analytical predictions for both strength and damage progression for a variety of geometries, lay-ups, thicknesses and notch sizes. However, intricate finite element analysis is required, and significant computational effort is needed to obtain convergence.

It is not wrong to say that the advanced composite structures are a challenge to analyze. This challenge is driven by the evolution of new advanced material systems and novel fabrication processes. Analytical procedures available in commercial linear/nonlinear finite element analysis tools for modeling these new material forms are often lagging behind the material science developments. When the materials technology matures sufficiently, verified analytical models can be developed, validated, and become available in the commercially available tools. It is also generally known that carbon–carbon composites consist of carbon fibers reinforced in a carbon matrix. Kostopoulos and Pappas [12] have adopted the damage zone model (DZM) (which is based on the physical intuition that a damage zone will form and grow in the stress-intense region of the specimen) to understand the fracture behavior of C/C composites containing a center circular hole.

This paper deals with the finite element analysis on 2D carbon–carbon laminates containing a central circular hole, utilizing ANSYS software package. Modifications are made in the point stress criterion to estimate the notched tensile strength of the laminates close to the test results of Kostopoulos and Pappas [12].

2. Analysis

Kostopoulos and Pappas [12] have generated the fracture data from the straight strip specimens having 225 mm long, 25.2 mm wide and 3 mm thick were cut from the 2D C/C composite plate, which was reinforced by orthogonally woven 8-harness satin fabric, stacked together in a symmetric (0°/90°) way. The fabric contained high tensile modulus fibres: modulus of elasticity = 94 GPa and Poisson's ratio = 0.07. $A_{11} = A_{22} = 20.172$ kN/mm; $A_{12} = 0.8681$ kN/mm and $A_{66} = 2.7072$ kN/mm are the orthotropic in-plane stiffnesses of the laminate. The corresponding axial, transverse and shear moduli as well as the Possion's ratio are: $E_{xx} = E_{yy} = 6.712$ GPa; $G_{xy} = 0.9024$ GPa and $\nu_{xy} = \nu_{yx} = 0.04303$. The un-notched strength of the laminate, $\sigma_0 = 173.2$ MPa. The bulk density of the material was 1.49 g/cm^3. Two acrylic plastic sheets were placed at both sides of the specimens to avoid delamination at the hole edge during drilling. All the defect (delamination) free specimens were tested on a closed-loop servo-hydraulic testing machine equipped with a hydraulic gripping system at room temperature in air.

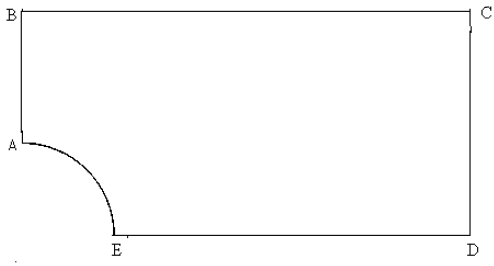

Figure 1. Quarter plate model of C–C composites with a central circular hole.

They conducted the static tensile tests using displacement control with a crosshead velocity of 0.1 mm/min.

Finite element analysis of the center-hole tensile specimens has been carried out utilizing the 8-noded quadrilateral plane stress element (PLANE183) of ANSYS software package for modeling a quarter portion of the specimen. Figure 1 shows the quarter plate model of an orthotropic thin 2D C/C composite plate having a central circular hole. Let O be the center of the hole. OA = OE = R, is the hole radius. OD = BC = 112.5 mm is half the length whereas OB = DC = 12.6 mm is half the width. The displacements U_x and U_y along X- and Y-axes are in the directions of length and width of the laminate. Because of symmetry, these are constrained along the edges AB and ED respectively. The edge CD along the X-direction is loaded with a tensile stress of 100 MPa. The stress concentration factor (K_{T}) is obtained from the finite element analysis by specifying the appropriate symmetric conditions, material properties and the applied tensile stress of 100 MPa. Figure 2 shows the stress contour plot for a quarter portion of a center-hole 2D carbon/carbon tensile specimen having 225 mm long, 25.2 mm wide, 3 mm thick and 1.5 mm open hole-diameter. The stress concentration factor for this configuration is worked out to be 3.94. The analysis results in Table 1 are found to be in reasonably good agreement with those obtained from the following empirical relation for the stress concentration factor (K_{T}) for anisotropic finite width plate containing central opening [7]:

$$\frac{K_{\mathrm{T}}^{\infty}}{K_{\mathrm{T}}} = \alpha + \frac{1}{2}\big(K_{\mathrm{T}}^{\infty} - 3\big)(1-\beta)\beta^{3}, \tag{1}$$

where $\alpha = 3(1 - D/W)\{2 + (1 - D/W)^3\}^{-1}$; $\beta = 1/2(\sqrt{9 - 8\alpha} - 1) D = 2R$, is the hole-diameter; W is the specimen width.

$$K_{\mathrm{T}}^{\infty} = 1 + \sqrt{2\left(\sqrt{\frac{E_{yy}}{E_{xx}}} - \nu_{yx}\right) + \frac{E_{yy}}{G_{xy}}}$$

$$= 1 + \sqrt{\frac{2}{A_{22}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)}$$

is the stress concentration factor for an infinite width orthotropic plate.

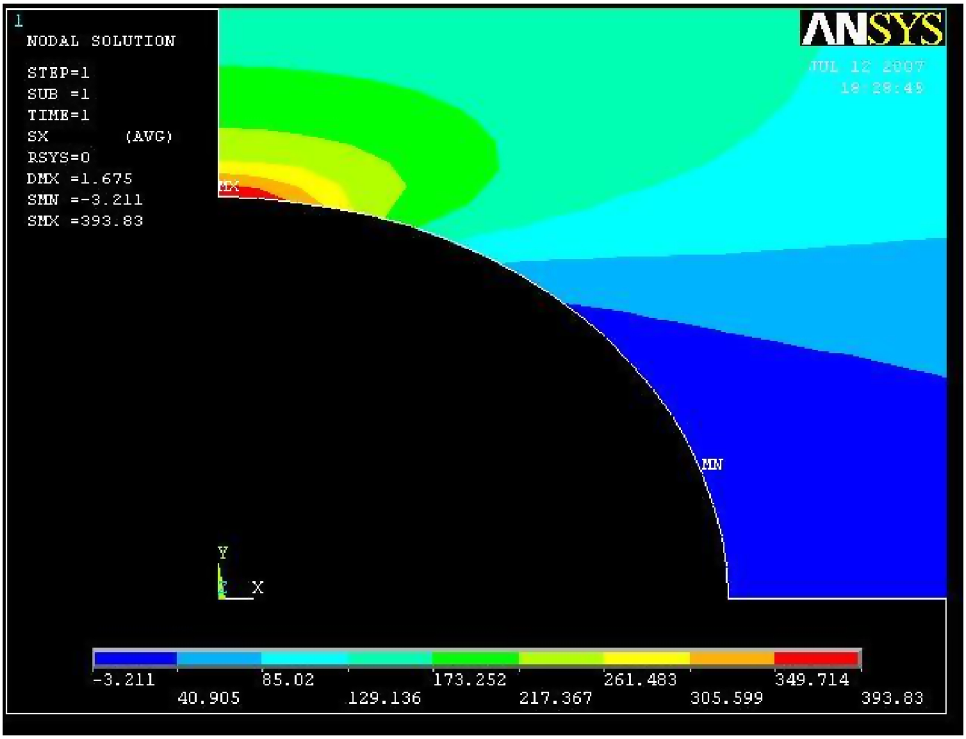

Figure 2. Stress contour plot of a quarter portion of a center-hole 2D carbon/carbon tensile specimen of dimensions 225 mm long, 25.2 mm wide, 3 mm thick with a 1.5 mm open hole-diameter. This figure is published in color in the online version.

Table 1.
Comparison of stress concentration factor and notched tensile strength for center-hole carbon/carbon composite specimens having 225 mm long, 25.2 mm wide and 3 mm thick

Hole diameter D $(=2R)$ (mm)	Stress concentration factor (K_T)		Notched tensile strength, σ_{Nh} (MPa)			σ_{Nh}^{∞} (MPa)	$\sigma_{Nh}^{\infty}/\sigma_0$	Characteristic length, a_{hp} (mm)
	Equation (1)	FEA	Test [12]	FEA equation (2)	Relative error (%)			
1.5	4.08	3.94	157.0	44.0	72.0	157.58	0.910	1.21
2.5	4.10	4.05	150.5	42.8	71.6	152.09	0.878	1.59
4.0	4.18	4.15	134.2	41.7	68.9	138.00	0.797	1.59
6.0	4.34	4.31	114.9	40.2	65.0	122.68	0.708	1.56
8.0	4.58	4.53	100.0	38.2	61.8	112.80	0.651	1.63
10.0	4.93	4.86	89.7	35.6	60.3	108.83	0.628	1.85

$E_{xx} = E_{yy} = 6.712$ GPa; $G_{xy} = 0.9024$ GPa; $\nu_{xy} = \nu_{yx} = 0.04303$; $K_T^{\infty} = 4.06$; $\sigma_0 = 173.2$ MPa.

The notched tensile strength (σ_{Nh}) is evaluated utilizing the un-notched strength (σ_0) of the material and the stress concentration factor (K_{T}) as:

$$\sigma_{\mathrm{Nh}} = \frac{\sigma_0}{K_{\mathrm{T}}}. \tag{2}$$

The notched tensile strength (σ_{Nh}) estimates in Table 1 are found to be highly conservative. The solutions assume that the laminate will remain complete and intact until the final failure occurs. Due to non-consideration of the stress evolution caused by the possibility of local matrix failures, the stress concentrations at the notch root will be overestimated, which produces notched strength predictions of cross-ply laminates to be lower than experimental values.

One of the stress fracture criteria of Whitney and Nuismer [6], known as the point stress criterion (PSC), states that fracture occurs when the stress (σ_y) ahead of the hole (see Fig. 4) at the characteristic distance (a_{hp}) is equal to the un-notched strength of the laminate (σ_0), i.e.,

$$\sigma_y(x,0) = \sigma_0 \quad \text{at } x = R + a_{\mathrm{hp}}, \tag{3}$$

where R is the hole-radius. The stress distribution adjacent to the hole in an infinite width plate under a uniform stress σ is given by [13]:

$$\sigma_y(x,0) = \sigma f\left(\frac{R}{x}\right), \quad x > R. \tag{4}$$

Here, $f(\xi) = 1/2\{2 + \xi^2 + 3\xi^4 - (K_{\mathrm{T}}^{\infty} - 3)(5 - 7\xi^2)\xi^6\}$.

The validity of equation (4) is verified in Fig. 3 by comparing the FEA results of 1.5 mm diameter hole in a tensile specimen having 225 mm long, 25.2 mm wide and 3 mm thick, which simulates the results of the hole in an infinite width plate under a uniform stress. Replacing K_{T}^{∞} in equation (4) by the stress concentration factor (K_{T}) for the finite width plate, the theoretical stress distribution ahead of the hole is obtained and compared with the finite element analysis results of ANSYS

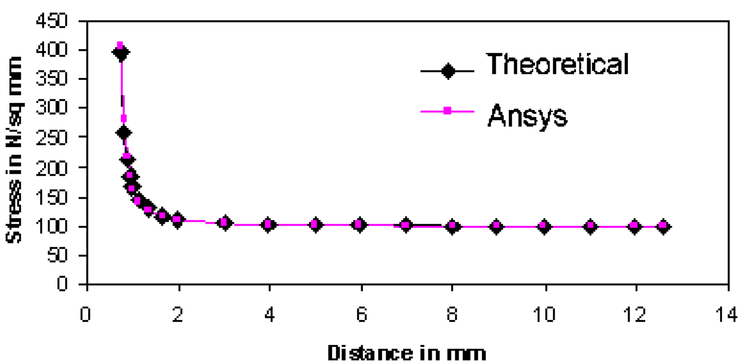

Figure 3. Stress distribution ahead of the 1.5 mm diameter hole in a thin orthotropic 2D carbon/carbon tensile specimen (225 × 25.2 × 3 mm). This figure is published in color in the online version.

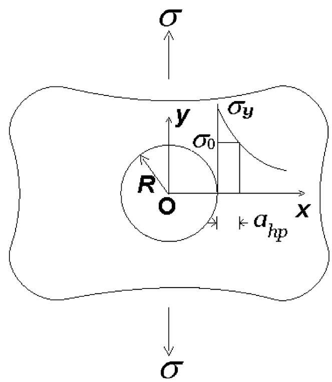

Figure 4. Characteristic length (a_{hp}) in a center-hole wide tensile panel.

for different hole-diameters of the 2D carbon/carbon laminates in Fig. 5. The theoretical stress distribution obtained from the empirical relation (4) is found to be in reasonably good agreement with the FEA results.

Applying the PSC to equation (4), the notched strength equation is obtained as:

$$\frac{\sigma_0}{\sigma_{\mathrm{Nh}}^{\infty}} = f\left(\frac{R}{R + a_{\mathrm{hp}}}\right). \tag{5}$$

The notched strength ($\sigma_{\mathrm{Nh}}^{\infty}$) of center circular hole wide tensile specimen is obtained from the experimental notched strength (σ_{Nh}) of the finite width tensile specimen by multiplying the correction factor ($K_{\mathrm{T}}^{\infty}/K_{\mathrm{T}}$). The characteristic length (a_{hp}) can be obtained from the fracture strength equation (5) using the Newton–Raphson iterative scheme.

It can be seen from the results in Table 1 that the characteristic length (a_{hp}) increases and the notched strength decreases with increase in the hole-diameter. Comparison of experimental data [8, 9, 14, 15] has also shown that characteristic lengths are material and hole-size-dependent. This calls for a modification in the point stress criterion [6] by writing a relation between the characteristic length (a_{hp}), the notched strength ($\sigma_{\mathrm{Nh}}^{\infty}$) of the wide center hole tensile specimens and the un-notched tensile strength of the laminate (σ_0) in the non-dimensional form as:

$$\frac{\sigma_0\sqrt{\pi a_{\mathrm{hp}}}}{K_{\mathrm{F}}} = 1 - m\frac{\sigma_{\mathrm{Nh}}^{\infty}}{\sigma_0}. \tag{6}$$

Here, $\sigma_0\sqrt{\pi a_{\mathrm{hp}}} = K_{\mathrm{Q}\infty}$, is the critical stress intensity factor, which is related to the notched strength ($\sigma_{\mathrm{Nh}}^{\infty}$) of the wide center hole tensile specimens and the un-notched tensile strength of the laminate (σ_0) through two fracture parameters K_{F} and m as being followed in metallic materials [16, 17]. The parameters K_{F} and m in equation (6) are to be determined by a least square curve fit to the data for a_{hp} and $\sigma_{\mathrm{Nh}}^{\infty}/\sigma_0$. For determination of these parameters, two notched specimen tests in addition to an un-notched specimen test are required; normally more tests are performed to take scatter in test results into account. It should be noted that $m = 0$

Hole-diameter, D = 4mm

Hole-diameter, D = 6mm

Hole-diameter, D = 8mm

Hole-diameter, D = 10mm

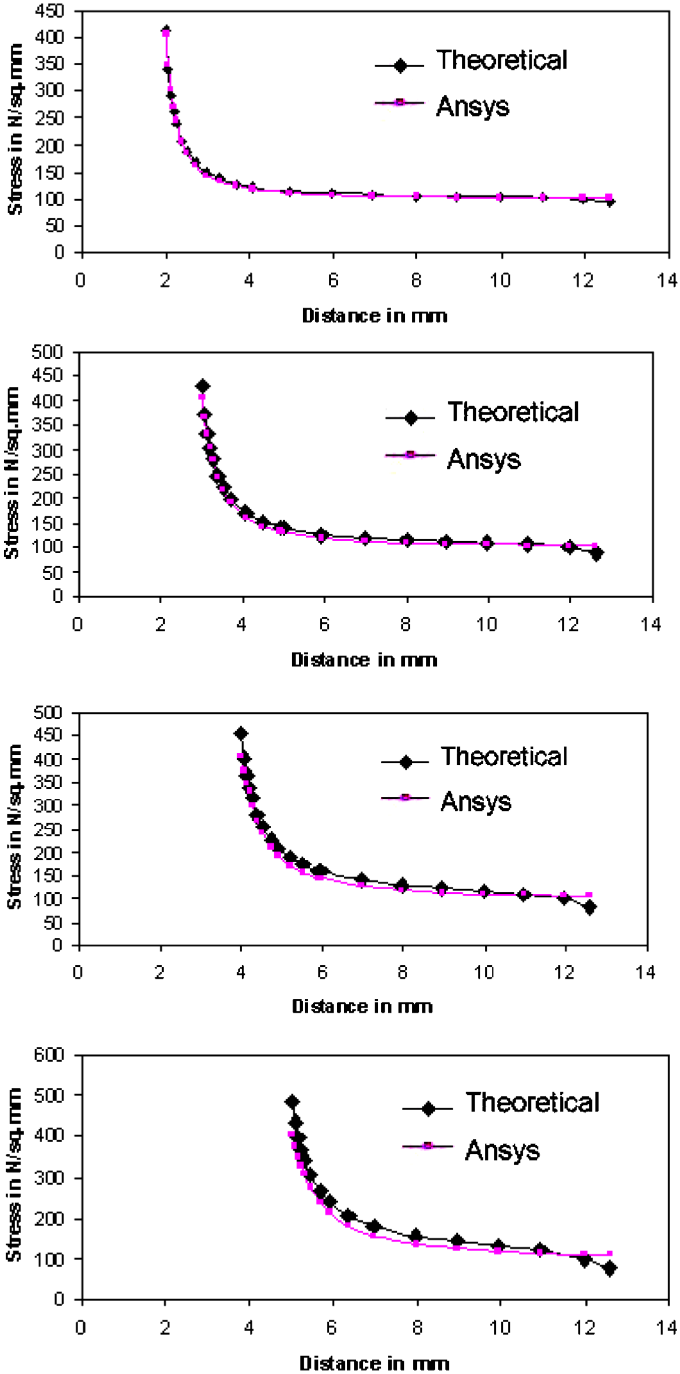

Figure 5. Comparison of theoretical and finite element analysis stress distribution ahead of the hole in a thin orthotropic 2D carbon/carbon tensile specimen (225 × 25.2 × 3 mm). This figure is published in color in the online version.

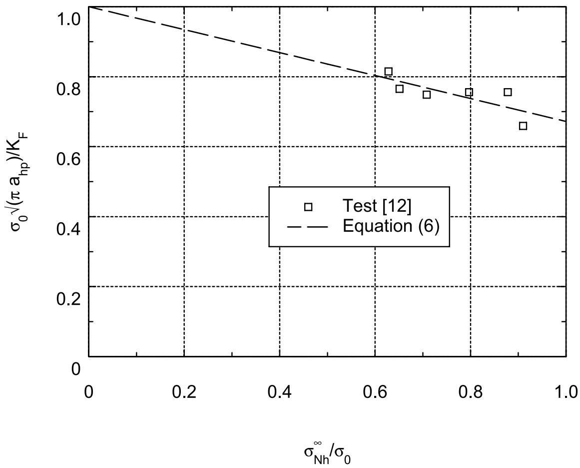

Figure 6. Validation of empirical relation (6) between the between the characteristic length (a_{hp}) and the fracture strength ($\sigma_{\mathrm{Nh}}^{\infty}$) through the fracture parameters K_{F} and m.

in equation (6) represents the case of constant damage size as per the original point stress criterion [6]. For the case: $m > 1$ and $\sigma_{\mathrm{Nh}}^{\infty} = \sigma_0$, equation (6) yields the result $\sigma_0\sqrt{\pi a_{\mathrm{hp}}}/K_{\mathrm{F}} < 0$. Hence, the variation of the parameter in equation (6) can be $0 \leqslant m \leqslant 1$. Whenever m is found to be greater than unity, the parameter m has to be truncated to 1 by suitably modifying the parameter K_{F} with the fracture data. If m is found to be less than zero, the parameter m has to be truncated to zero and the average of $\sigma_0\sqrt{\pi a_{\mathrm{hp}}}$ values from the fracture data yields the parameter K_{F}. Figure 6 shows the validation of the empirical relation (6) between the characteristic length (a_{hp}) and the fracture strength ($\sigma_{\mathrm{Nh}}^{\infty}$) through the fracture parameters K_{F} and m.

Equation (6) is used in equation (5) for eliminating the characteristic length (a_{hp}) and the resulting non-linear fracture strength equation is solved using the Newton–Raphson iterative scheme to obtain the notched strength ($\sigma_{\mathrm{Nh}}^{\infty}$) of the infinite width plate for the specified notch size. The notched tensile strength (σ_{Nh}) of the finite width plate is then obtained by dividing $\sigma_{\mathrm{Nh}}^{\infty}$ with the correction factor ($K_{\mathrm{T}}^{\infty}/K_{\mathrm{T}}$). Table 2 gives the parameters K_{F} and m and the notched tensile strength of center-hole carbon/carbon composites from the modified point stress criterion. The notched tensile strength estimates are found to be within 2% of test results [12].

Tercan *et al.* [9] provided empirical relations for the characteristic lengths in terms of notch size and specimen width. The material constants in the empirical relations are found to vary with the specimen width. These empirical relations demand testing of the intended width specimens. Such a requirement is not essential, when the inherent flaw length is expressed in terms of notched and un-notched strength of the material as in equation (6). To examine this aspect, fracture data on weft-knitted glass fiber composite plates with different central holes and different plate widths generated by Tercan *et al.* [9] are considered. The knitted fabrics used are fabricated from 200 tex glass yarn with epoxy resin and cured in a hot press

Table 2.
Comparison of notched tensile strength estimates with test data of center-hole carbon/carbon composite specimens having 225 mm long, 25.2 mm wide and 3 mm thick $E_{xx} = E_{yy} = 6.712$ GPa; $G_{xy} = 0.9024$ GPa; $\nu_{xy} = \nu_{yx} = 0.04303$; $K_T^\infty = 4.06$; $\sigma_0 = 173.2$ MPa; $K_F = 16.2$ (MPa$\sqrt{m}$); $m = 0.328$

Hole diameter D (= $2R$) (mm)	K_T equation (1)	Fracture strength, σ_{Nh} (MPa)				
		Test [12]	Damage zone model [12]	Relative error (%)	Modified point stress criterion (present analysis)	Relative error (%)
1.5	4.08	157.0	159.13	−1.4	159.28	−1.5
2.5	4.10	150.5	148.69	1.2	147.68	1.9
4.0	4.18	134.2	134.64	−0.3	132.92	1.0
6.0	4.34	114.9	119.44	−4.0	116.35	−1.3
8.0	4.58	100.0	106.81	−6.8	101.67	−1.7
10.0	4.93	89.7	95.25	−6.2	88.23	1.6

machine. Notched tensile specimens were tested in wale as well as in course directions. Fracture parameters (K_F and m) in equation (6) are evaluated considering the fracture data of 20 mm width specimens, which were loaded in wale direction, and 10 mm width specimens, which were loaded in course direction. Material properties and fracture parameters in wale direction are: $E_{xx} = 7.25$ GPa; $E_{yy} = 6.75$ GPa; $G_{xy} = 1.69$ GPa; $\nu_{xy} = 0.23$; $\sigma_0 = 128$ MPa; $K_F = 13.152$ MPa$\sqrt{m}$ and $m = 1.0$. Material properties and fracture parameters in course direction are: $E_{xx} = 6.75$ GPa; $E_{yy} = 7.25$ GPa; $G_{xy} = 1.69$ GPa; $\nu_{xy} = 0.214$; $\sigma_0 = 135$ MPa; $K_F = 14.781$ MPa$\sqrt{m}$; $m = 0.969$. Table 3 gives comparison of notched tensile strength estimates with the test data of Tercan *et al.* [9]. The fracture strength estimates were found to be with in ±10% of the test results.

Dirikolu and Aktas [14] have generated the fracture data on $(0°)_3$ carbon/epoxy composite plates (110 mm length, 25 mm width and 0.89 mm thickness) containing 4, 6 or 8 mm diameter holes. The tensile properties of the material are: $E_{xx} = 29$ GPa; $E_{yy} = 27$ GPa; $G_{xy} = 2.03$ GPa; $\nu_{xy} = 0.25$ and $\sigma_0 = 405$ MPa. Average values of the notched tensile strength (σ_{Nh}) obtained from tests for the composite plates having 4, 6 and 8 mm diameter holes are [14]: 191, 168 and 152 MPa, respectively. The parameters K_F and m in equation (6) are: $K_F = 28.90$ MPa$\sqrt{m}$ and $m = 1$. The notched strength (σ_{Nh}) estimates for the composite plates having 4, 6 and 8 mm diameter holes obtained from the present modified point stress criterion are: 196.4, 169.2 and 147.4 MPa, respectively. The notched tensile strength estimates were found to be within ±3% of the test results [14].

Lawcock *et al.* [15] investigated the residual strength of carbon fiber reinforced metal laminates with center circular holes of various sizes. The stress concentration factor, $K_T^\infty = 3$, for the quasi-isotropic wide composite plate with center circular hole. Table 4 gives the comparison of fracture strength estimates with test data [15].

Table 3.
Comparison of notched tensile strength (σ_{Nh}) estimates of weft-knitted glass fiber composite plates

Hole diameter D (= $2R$) (mm)	Notched tensile strength, σ_{Nh} (MPa)					
	Loaded in wale direction			Loaded in course direction		
	Test [9]	Present analysis	Relative error (%)	Test [9]	Present analysis	Relative error (%)
Specimen width, $W = 10$ mm						
2	72.0	78.41	−8.9	86.0	85.67	0.4
3	63.2	68.84	−8.9	73.0	75.27	−3.1
4	55.3	59.44	−7.5	64.0	64.98	−1.5
5	47.0	49.90	−6.2	56.7	54.49	3.9
Specimen width, $W = 20$ mm						
2	80.7	81.19	−0.6	93.5	88.71	5.1
3	73.0	74.84	−2.5	84.0	81.85	2.6
4	70.0	69.60	0.6	76.3	76.16	0.2
5	66.3	64.88	2.1	79.0	71.02	10.1
Specimen width, $W = 40$ mm						
2	83.8	81.85	2.3	85.0	89.43	−5.2
3	77.0	76.27	1.0	82.4	83.41	−1.2
4	78.0	72.07	7.6	82.0	78.86	3.8
5	75.0	68.63	8.5	81.0	75.13	7.3

Table 4.
Comparison of notched tensile strength, σ_N (MPa) of carbon fiber reinforced metal laminates with center circular-holes of various sizes

Hole diameter D (= $2R$) (mm)	Notched tensile strength, σ_{Nh} (MPa)		
	Test [15]	Modified point stress criterion	Relative error (%)
10	480.3	492.0	−2.4
20	421.9	406.9	3.6
30	338.3	346.3	−2.4
40	292.2	289.2	0.9

$W = 90$ mm; $K_T^{\infty} = 3$; $\sigma_0 = 883.4$ MPa; $K_F = 122.8$ MPa$\sqrt{m}$; $m = 0.862$.

The scatter in the test data of central circular open-hole specimens is found to be within ±5% of its average value. The notched strength estimates are found to be within ±4% of the test results [15].

3. Conclusions

Understanding failure in composites is complicated by the variety of damage mechanisms that can occur in a laminate under loading. When a stress is applied to composite, different types of damage can occur that may lead to failure of the laminate or simply cause local redistribution of stresses. Finite element analysis (FEA) has been carried out on center-hole 2D carbon/carbon (C/C) laminates under tension utilizing ANSYS software package.

For orthotropic thin 2D C/C composite plates containing a center circular hole, the eight noded quadrilateral plane stress element (PLANE 183) of ANSYS is utilized. Existing empirical relations obtained from the 2D elasticity solutions for the normal stress distribution ahead of the hole for the infinite width specimen and the stress concentration factor for finite width specimens were found to be in good agreement with the present FEA results. Hu *et al.* [18] have performed a very fine 3D finite element analysis of a $[0°/90°]_S$ specimen and found good agreement with the 2D model for the in-plane stress components. Other studies [19–24] involving 3D models have almost exclusively been concerned with interlaminar stresses. It is known that the traditional displacement-interpolation-based finite elements are not able to ensure the continuity of interlaminar stresses. To overcome this deficiency, interface elements will be used between 3D elements belonging to layers of different orientations. The presence of interface elements does not affect the layer stresses and enhances the computational time.

Notched tensile strength estimates utilizing the stress concentration factor and the un-notched tensile strength of the 2D carbon/carbon laminates are found to be highly conservative when compared to those of test results. This indicates the possibility of local redistribution of the stresses ahead of the hole in composites taking place prior to failure. Modifications made in the point stress criterion yield the notched strength estimates close to the test results of different composite materials. Finite element analysis is required to obtain the stress distribution for any complex configuration containing cut-outs under complex loading conditions and the notched strength of such configurations can be easily estimated following the modified point stress criterion. For the case of notched biaxial loaded structures, the proposed numerical analysis can be easily applied using the un-notched strength and the stress distribution ahead of the notch in the structure under biaxial loading conditions.

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